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Plane Waves Propagating in Gases Composed of Composite Particles

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The quantum discrete kinetic equations are solved to study the propagation of plane waves in a system of composite particles with hard-sphere interactions and the filling factor (ν) being 1/2. We compare the dispersion relations thus obtained by the relevant Pauli-blocking parameter B which describes the different-statistics particles for the quantum analog of the discrete Boltzmann system when B is positive (Bose gases), zero (Boltzmann gases), and negative (Fermi Gases). We found, as the effective magnetic field being zero ($\nu = 1/2$ using the composite fermion formulation), the electric field effect will induce anomalous dispersion relations.

KEY WORDS: Pauli-blocking effect; external field.

1. INTRODUCTION

The progress in semiconductor technology has opened a rich field of studies focused on the fundamental electron–electron interactions and quantum effects in artificial atoms and molecules (composite particles Ghirardi and Marinatto, 2004; Avancini and Krein, 1995; Harju *et al.*, 2002; Braun and Vechernin, 2004; Kuze and Sirois, 2003; Yabu *et al.*, 2004; Schrieffer, 2004; Agop *et al.*, 2003; Ichinose and Matsui, 2003; Fradkin *et al.*, 1998). The most striking feature of two-dimensional semiconductor quantum dots (QD) and quantum dot molecules (QDM) is that the correlation and magnetic field effects are greatly enhanced compared with their normal counterparts. Meanwhile, the (composite-particle) system parameters can easily be changed, unlike in real atoms and molecules where the parameters are natural constants. The controllable parameters make it possible to tailor the semiconductor structures and, for example, to switch between different ground states (Harju *et al.*, 2002). In addition to the interesting and fundamental correlation and quantum effects, this system is very important as

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a candidate for the gate of a quantum computer (Burkard *et al.*, 1999; Hu and Das Sarma, 2000).

In the presence of a strong magnetic field $\mathcal{B}$ transverse to a two-dimensional system of electrons, the tiny cyclotron orbits of an electron are quantized to produce discrete kinetic energy levels, called *Landau levels*. The degeneracy of each Landau level-that is to say, its maximum population per unit area is $\mathcal{B}/\phi_0$, where $\phi_0 = h/e$ is the elementary quantum of magnetic flux. This degeneracy implies that the number of occupied Landau levels, called the filling factor, is $\nu = \rho\phi_0/\mathcal{B}$, where ρ is the two-dimensional electron density. The crucial point is that the many-particle ground state of electrons at $\nu < 1$ was highly degenerate in the absence of interaction, with all lowest Landau level configurations having the same energy.

It is well established that the electron–electron interaction strongly affects the compressibility of a two-dimensional electron system at zero magnetic field and even leads to the negative sign of the compressibility. Surprisingly, Dorozhkin *et al.* found that its contribution is almost identical for the metallic states of electrons at zero magnetic field and of composite fermions at Landau level filling 1/2 (Dorozhkin *et al.*, 2002).

The fractional quantum Hall effect (FQHE) is observed in high-mobility two-dimensional electron systems in the low-temperature, high-magnetic-field regime (Jobst *et al.*, 2000). It is believed that the FQHE arises from strong electron–electron interactions, causing the two-dimensional electrons to condense into a fractional quantum Hall liquid. Jain introduced the concept of "composite fermion" (CF) where each electron is bound to two magnetic flux quanta, and in this picture the FQHE can then be understood as a manifestation of the integer quantum Hall effect of composite fermions (Jain *et al.*, 2000). At a Landau level filling factor $\nu = 1/2$, a two-dimensional electron system can be transformed into a composite fermion system interacting with a Chern–Simons gauge field (Heinonen, 1998). To date, a wide variety of experiments have demonstrated that at $\nu = 1/2$ the effective magnetic field acting on the composite fermions is zero (Heinonen, 1998). Away from $\nu = 1/2$, the effective magnetic field acts on the composite fermions is given by $\mathcal{B}_{\text{eff}} = \mathcal{B}_{\text{ext}} - \mathcal{B}_{(\nu=1/2)}$ where $\mathcal{B}_{\text{ext}}$ is the applied external magnetic field. At $\nu = 1/2$, each electron is bound to two magnetic flux quanta, and thus the density of the composite fermion system is equal to that of the electron system.

A semiclassical theory based on the Boltzmann transport equation for a two-dimensional electron gas modulated along one direction with weak electrostatic or magnetic modulations have been proposed (Zwerschke and Gerhardts, 2001; Ustinov and Kravtsov, 1995; Zwerschke and Gerhardts, 1998; Zimbovskaya, 2003). Ustinov and Kravtsov studied the giant magnetoresistance effect in magnetic superlattices for the current perpendicular to and in the layer planes within a unified semiclassical approach that is based on the Boltzmann equation with exact boundary conditions for the spin-dependent distribution functions of electrons.

Interface processes responsible for the magnetoresistance were found to be different in these geometries, and that can result in an essential difference in general behaviour between the in-plane magnetoresistance and the perpendicular-plane one (Ustinov and Kravtsov, 1995).

Boltzmann's equation provides an adequate starting point of transport calculations for two-dimensional electron systems in the presence of periodic electric and magnetic modulation fields, both in the regime of the low-field positive magnetoresistance and of the Weiss oscillations at intermediate values of the applied magnetic field. For example, Zwerschke and Gerhardts solved Boltzmann's equation by the method of characteristics, which allows to exploit explicitly information about the structure of the phase space. That structure becomes very complicated if the amplitudes of the modulation fields become so large and the average magnetic field becomes so small that, in addition to the drifting cyclotron orbits, channeled orbits exist and drifting cyclotron orbits extend over many periods of the modulation (Zwerschke and Gerhardts, 1998, 2001).

Zwerschke and Gerhardts (1998, 2001) considered the 2DEG in the x–y plane as a degenerate Fermi gas, with Fermi energy $E_F = m^* v_F^2/2$, of (non-interacting) particles with effective mass m^* and charge $-e$ obeying classical dynamics, i.e. Newton's equation $m^*\dot{\mathbf{v}} = -e[\mathbf{E}_e + (\mathbf{v} \times \mathcal{B})]$. In equilibrium, the electric field is given by $\mathbf{E}_e(\mathbf{r}) = \nabla V(\mathbf{r})/e$, where $V(\mathbf{r})$ is the modulating electrostatic potential. In thermal equilibrium, all states with energy below E_F are occupied, and for the linear response to an external homogeneous electric field $\mathbf{E}_0$ only the electrons with energy $E(\mathbf{r}, \mathbf{v}) = (m^* \mathbf{v}^2)/2 + V(\mathbf{r}) = E_F$ contribute to the current. The distribution function $f(\mathbf{r}, \mathbf{v}, t)$ obeys the Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathcal{D}f - \mathcal{C}[f; \mathbf{r}, \mathbf{v}] = \mathbf{v} \cdot \mathbf{E}_0,$$

where the drift term $\mathcal{D}$ describes the change due to the natural motion of the electrons in the modulation field (in absence of $\mathbf{E}_0$), and $\mathcal{C}$ is the collision operator. We might use polar coordinates in the velocity space, $\mathbf{v} = v\mathbf{u}$ with $v(\mathbf{r}) = v_F[1 - V(\mathbf{r})/E_F]^{1/2}$ and $\mathbf{u}(\Theta)$=(cos Θ, sin Θ). Sometimes (Avancini and Krein, 1995), the drift term reads $\mathcal{D} = \mathbf{v} \cdot \nabla + [\omega_c + \omega_{el}(\mathbf{r}, \Theta)]\partial/\partial\Theta$, with cyclotron frequency $\omega_c = e\mathcal{B}_{eff}/m^*$ and $\omega_{el}(\mathbf{r}, \Theta) = (\nabla V)\mathbf{t}$ with $\mathbf{t}(\Theta) = (\sin\Theta, \cos\Theta)$.

Recently Jobst investigated the magnetoresistance of a weakly density modulated high mobility two-dimensional electron system around filling factor $\nu = 1/2$ (Jobst *et al.*, 2000). The experimental ρ_{xx}-traces around $\nu = 1/2$ were well described by novel model calculations, based on a semiclassical solution of the Boltzmann equation, taking into account anisotropic scattering. We also noticed that, the effects of a tunable periodic density modulation imposed upon a 2D electron system have been probed using surface acoustic waves by Willett *et al.* (1998), Willett and Pfeiffer (1996). A substantial effect was induced at filling factor 1/2 in which the Fermi surface properties of the CF are anisotropically replaced by features similar to those seen in quantum Hall states. The response measured using different SAW wavelengths and similarities in the temperature dependence between the modulation induced features at 1/2 and quantum Hall states were described therein (Willett *et al.*, 1998; Willett and Pfeiffer, 1996).

Motivated by the interesting issues about $\nu = 1/2$, we like to study their characteristics relevant to the sound propagation in CF gases here using our verified quantum (discrete) kinetic approaches (Chu, 2002, 2003, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b). In the discrete kinetic model approach (Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991), the main idea is to consider that the particle velocities belong to a given finite set of velocity vectors, e.g., $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_p$, p is a finite positive integer. Only the velocity space is discretized, the space and time variables are continuous (Chu, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b) (please see the detailed references therein). By using the discrete velocity model approach, the velocity of propagation of plane waves can be classically determined by looking for the properties of the solution of the conservation equation referred to the equilibrium state.

As a continuous attempt of plane waves propagating in dilute gases (Chu, 1999a,b, 2002a, 2003), considering the quantum analog of the discrete kinetic model and the Uehling–Uhlenbeck collision term which could describe the collision of a gas of dilute hard-sphere Fermi-, Boltzmann- or Bose-particles by tuning a parameter θ (Chu, 2004a; Vedenyapin *et al.*, 1995; Uehling and Uhlenbeck, 1933) (via a *Pauli-blocking factor* of the form $1 + \theta f$ with f being a normalized distribution function giving the number of particles per cell, say, a unit cell, in phase space), in this paper, we plan to study the dispersion relations of plane ultrasonic waves propagating in composite-particle gases by the quantum discrete kinetic model which has been verified before. This presentation will give more clues to the studies of the quantum wave dynamics in a system composed of composite particles under strong external fields (Yabu *et al.*, 2004).

2. THEORETICAL FORMULATIONS

The gas is presumed to be composed of identical hard-sphere particles of the same mass. The discrete number density (of particles) is denoted by $N_i(\mathbf{x}, t)$ associated with the velocity $\mathbf{u}_i$ at point $\mathbf{x}$ and time t. Following the CF model, around $\nu = 1/2$ or any even-denominator $\nu = 1/2p$, $2p$ fictitious magnetic flux quanta ($\phi_0 = h/e$) are attached to each electron in the direction opposite to the external magnetic field B. The so formed composite particles follow Fermi statistics and are named composite fermions. The flux attachment transforms the strongly interacting two-dimensional electron system (2DES) of density ρ in a high a magnetic field into an equivalent weakly interacting CF system, which experiences a smaller effective magnetic field, $\mathcal{B}_{\text{eff}} = \mathcal{B} - 2\rho p \phi_0$. In particular, at exact even-denominator fillings, $\nu = 1/2p, \mathcal{B} = 2p\,\rho\,h/e = 2\rho p\,\phi_0$, and $\mathcal{B}_{\text{eff}}$ vanishes. Under these conditions, the CFs reside in a magnetic field-free region and, like ordinary 2D electrons at $\mathcal{B} = 0$, they form a Fermi sea. The particles of this Fermi system have an effective mass which is of purely electron–electron interaction origin and therefore proportional to $e^2/\epsilon\, l_B$ (e.g., ϵ is the dielectric constant of GaAs and $l_B = (\hbar c/e\mathcal{B})^{1/2}$ is the magnetic length).

If only nonlinear binary collisions and the effective magnetic field $\mathbf{B}_{\text{eff}}$ being zero (for $\nu = 1/2$ in the CF sense) are considered, we have for the evolution of N_i,

$$\frac{\partial N_i}{\partial t} + \mathbf{u}_i \cdot \nabla N_i - \frac{e\mathbf{E}}{m^*} \cdot \nabla_{\dot{\mathbf{u}}_i} N_i = \mathcal{C}_i$$

$$\equiv \sum_{j=1}^{p} \sum_{(k,l)} \left(A_{kl}^{ij} N_k N_l - A_{ij}^{kl} N_i N_j \right), \quad i = 1, \ldots, p, \tag{1}$$

where $\mathbf{E}$ is the electric field, m^* is the effective mass of the particle, (k, l) are admissible sets of collisions (Chu, 2002, 2003, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b). We may also define the right-hand side of above equation as

$$\mathcal{C}_i(N) = \frac{1}{2} \sum_{j,k,l} \left(A_{kl}^{ij} N_k N_l - A_{ij}^{kl} N_i N_j \right), \tag{2}$$

with $i \in \Lambda = \{1, \ldots, p\}$, and the summation is taken over all $j, k, l \in \Lambda$, where A_{kl}^{ij} are nonnegative constants satisfying (Chu, 2002, 2003, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b) (i) $A_{kl}^{ji} = A_{kl}^{ij} = A_{lk}^{ij}$: *indistinguishability of the particles in collision*, (ii) $A_{kl}^{ij}(u_i + u_j - u_k - u_l) = 0$: *conservation of momentum in the collision*, (iii) $A_{kl}^{ij} = A_{ij}^{kl}$: *microreversibility condition*. The conditions defined for discrete velocities above are valid for elastic binary collisions such that momentum and energy are preserved. The collision operator is now simply obtained by joining A_{ij}^{kl} to the corresponding transition probability densities a_{ij}^{kl} through $A_{ij}^{kl} = S|\mathbf{u}_i - \mathbf{u}_j|a_{ij}^{kl}$, where,

$$a_{ij}^{kl} \geq 0, \quad \sum_{k,l=1}^{p} a_{ij}^{kl} = 1, \quad \forall\, i, j = 1, \ldots, p;$$

with S being the effective collisional cross-section (Chu, 2002, 2003, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b). If all n ($p = 2n$) outputs are assumed to be equally probable, then $a_{ij}^{kl} = 1/n$ for all k and l, otherwise $a_{ij}^{kl} = 0$. Collisions which satisfy the conservation and reversibility conditions which have been stated above are defined an *admissible collision* (Chu, 2002, 2003, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b).

With the introduction of the Uehling-Uhlenbeck collision term (Chu, 2004a; Vedenyapin *et al.*, 1995; Uehling and Uhlenbeck, 1933) in Eq. (1) or Eq. (2),

$$\mathcal{C}_i = \sum_{j,k,l} A_{kl}^{ij} \, [N_k N_l (1 + \theta N_i)(1 + \theta N_j) - N_i N_j (1 + \theta N_k)(1 + \theta N_l)], \qquad (3)$$

for $\theta < 0$ we obtain a gas of Fermi-particles; for $\theta > 0$ we obtain a gas of Bose-particles, and for $\theta = 0$ we obtain Eq. (1).

From Eq. (3), the model of quantum discrete kinetic equation for dilute hard-sphere gases proposed before (Chu, 2004a; Vedenyapin *et al.*, 1995; Uehling and Uhlenbeck, 1933) is then a system of $2n(= p)$ semilinear partial differential equations of the hyperbolic type:

$$\frac{\partial}{\partial t} N_i + \mathbf{v}_i \cdot \frac{\partial}{\partial \mathbf{x}} N_i - \frac{e\mathbf{E}}{m^*} \cdot \nabla_{\mathbf{v}_i} N_i = \frac{cS}{n} \sum_{j=1}^{2n} N_j N_{j+n} (1 + \theta N_{j+1})(1 + \theta N_{j+n+1})$$
$$-2cS N_i N_{i+n} (1 + \theta N_{i+1})(1 + \theta N_{i+n+1}), \quad (4)$$

where $N_i = N_{i+2n}$ are unknown functions, and $\mathbf{v}_i = c(\cos[(i-1)\pi/n], \sin[(i-1)\pi/n]), i = 1, \ldots, 2n$; c is a reference velocity modulus (Chu, 2002, 2003, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991; Chu, 1999a,b). The admissible collisions as $n = 2$ are $(\mathbf{v}_1, \mathbf{v}_3) \longleftrightarrow (\mathbf{v}_2, \mathbf{v}_4)$.

We notice that the right-hand side of the Eq. (4) is highly nonlinear and complicated for a direct analysis. As passage of the sound wave causes a small departure from an equilibrium resulting in energy loss owing to internal friction and heat conduction (Kneser, 1961; Grad, 1966), we linearize above equations around a uniform equilibrium state (N_0) by setting $N_i(t, x) = N_0\,(1 + P_i(t, x))$, where P_i is a small perturbation. The equilibrium here is presumed to be the same as in Chu (2004a), Vedenyapin *et al.* (1995), Uehling and Uhlenbeck (1933) (in the absence of applied fields, the electrons will be at equilibrium and the distribution function will be the equilibrium distribution function $N_0(\epsilon - \mu_0) = [1 + \exp(\epsilon - \mu_0)/k_B T)]^{-1}$, where μ_0 is the chemical potential, k_B is the Boltzmann constant, the corresponding Fermi surface is defined by the equations $\epsilon(\mathbf{k}) = \mu_0$ in the quasi-momentum space, $\mathbf{k}$ is the wave vector). After some similar manipulations as mentioned in Chu (1999a,b, 2002a, 2003), with $B = \theta N_0$ (Chu, 2002, 2004a; Vedenyapin *et al.*, 1995; Uehling and Uhlenbeck, 1933), which gives or defines the (proportional) contribution from dilute Bose gases (if $\theta > 0$, e.g., $\theta = 1$), or dilute Fermi gases (if $\theta < 0$, e.g., $\theta = -1$), we then have

$$\left[\frac{\partial^2}{\partial t^2} + c^2 \cos^2 \frac{(m-1)\pi}{n} \frac{\partial^2}{\partial x^2} + 4cSN_0(1+B)\frac{\partial}{\partial t}\right] D_m$$
$$-\frac{4cSN_0(1+B)}{n} \sum_{k=1}^{n} \frac{\partial}{\partial t} D_k = \text{RHS}, \qquad (5)$$

where $D_m = (P_m + P_{m+n})/2, m = 1, \ldots, n$, since $D_1 = D_m$ for $1 = m \pmod{2n}$. Here, RHS denotes the contribution from the electric field. This term could be worked out by following the previous approaches (Ustinov and Kravtsov, 1995; Butler *et al.*, 2000; Noce and Cuoco, 2000) (cf. the second term in the left-hand side of the Eq. (4) in Ustinov and Kravtsov, 1995).

We are ready to look for the solutions in the form of plane wave $D_m = d_m \exp i(kx - \omega t)$, $(m = 1, \ldots, n)$, with $\omega = \omega(k)$. This is related to the dispersion relations of (forced) plane waves propagating in dilute (monatomic) hard-sphere Bose $(B > 0)$ or Fermi $(B < 0)$ gases. So we have

$$\left(1 + ih(1+B) - 2\lambda^2 \cos^2 \frac{(m-1)\pi}{n}\right) d_m - \frac{ih(1+B)}{n} \sum_{k=1}^{n} d_k = \text{RHS},$$
$$\times\, m = 1, \ldots, n, \tag{6}$$

with

$$\lambda = \frac{kc}{\sqrt{2}\omega}, \qquad h = \frac{4cSN_0}{\omega},$$

where λ is complex and $h\,(\propto 1/Kn)$ is the rarefaction parameter of the Bose- or Fermi-particle gas (Kn is the Knudsen number which is defined as the ratio of the mean free path of Bose or Fermi gases to the wave length of the plane (sound) wave).

We firstly consider the case of rather weak electric field. It means RHS ≈ 0 considering other domainted terms in the Eq. (6). Let $d_m = \mathcal{C}/(1 + ih(1+B) - 2\lambda^2 \cos^2[(m-1)\pi/n])$, where $\mathcal{C}$ is an arbitrary, unknown constant, since we here only have interest in the eigenvalues of above relation. The eigenvalue problems for different $2 \times n$-velocity model reduces to

$$1 - \frac{ih(1+B)}{n} \sum_{m=1}^{n} \frac{1}{1 + ih(1+B) - 2\lambda^2 \cos^2\,(m-1)\pi/n} = \text{RHS} \sim 0. \tag{7}$$

We solve only $n = 2$ case, i.e., 4-velocity case since for $n > 2$ there might be spurious invariants (Chu, 2004a; Platkowski and Illner, 1988; Bellomo and Gustafsson, 1991). For 2×2-velocity model, we obtain

$$1 - \left[\frac{ih(1+B)}{2}\right] \sum_{m=1}^{2} \left\{\frac{1}{[1 + ih(1+B) - 2\lambda^2 \cos^2 (m-1)\pi/2]}\right\} = 0.$$

3. RESULTS AND DISCUSSIONS

With the filling factor $\nu = 1/2$, we are now ready to obtain the dispersion relations for plane (sound) waves propagating in composite-paritcle gases (with

$\mathcal{B}_{\rm eff} = 0$). By using the standard symbolic or numerical software, we can obtain the complex roots ($\lambda = \lambda_r + i\,\lambda_i$) from the polynomial equation above. The roots are the values for the nondimensionalized dispersion (positive real part; a relative measure of the sound or phase speed) and the attenuation or absorption (positive imaginary part), respectively.

Calculated results (for zero external field) follow the conventional dispersion relations of ultrasound propagation in dilute hard-sphere gases (Chu, 1999a,b, 2002a, 2003; Stamper-Kurn and Miesner, 1998; Andrews and Stamper-Kurn, 1998; Lee *et al.*, 1957; Lee and Yang, 1957). Our results show that as $|B|$ (B: the Pauli-blocking parameter) increases, the dispersion (λ_r) will reach the continuum or hydrodynamical limit ($h \to \infty$) earlier. The phase speed of the plane (sound) wave in Bose gases (even for small but fixed h) increases more rapid than that of Fermi gases (w.r.t. to the standard conditions : $h \to \infty$) as the relevant parameter B increases. For all the rarefaction measure (h), plane waves propagate faster in Bose-particle gases than Boltzmann-particle and Fermi-particle gases. Meanwhile, the maximum absorption (or attenuation) for all the rarefaction parameters h keeps the same for all B. There are only shifts of the maximum absorption state (defined as $h_{\rm max}$) w.r.t. the rarefaction parameter h when B increases. It seems for the same mean free path or mean collision frequency of the dilute hard-sphere gases (i.e., the same h as h is small enough but $h < h_{\rm max}$) there will be more absorption in Bose particles than those of Boltzmann and Fermi particles when the plane (sound) wave propagates (Chu, 2004a).

In contrast, for the same h (as h is large enough but $h > h_{\rm max}$, there will be less absorption in Bose particles than those of Boltzmann particles when the plane wave propagates. When B (i.e., θ) is less than zero or for the Fermi-particle gases, the resulting situations just mentioned above reverse. For instance, as the rarefaction parameter is around 10, which is near the hydrodynamical or continuum limit, we can observe that the ultrasound absorption becomes the largest when the plane (sound) wave propagates in hard-sphere Fermi gases. That in Bose gases becomes the smallest. As for cases of dilute Fermi gases ($B < 0$), the rather small dispersion value (relative measure of different phase speeds between the present rarefied state : h and the hydrodynamical state : $h \to \infty$) when B approaches to -1 perhaps means there is the Fermi pressure which causes a Fermi gas to resist compression (Chu, 2002, 2003, 2004b).

If there is no rarefaction effect ($h = 0$), we have only real roots for all the models. Once $h \neq 0$, the imaginary part appears and the spectra diagram for each gas looks entirely different. In short, the dispersion ($k_r c/(\sqrt{2}\omega)$) reaches a continuum-value of 1 (or saturates) once h increases to infinity. We noticed that the increasing trend for the expression of our dispersion (λ_r; dimensionless) when waves propagating in Bose gases is similar to that (of dimensional sound speed) reported in Stamper-Kurn and Miesner (1998), Andrews and Stamper-Kurn (1998), Lee *et al.* (1957) Lee and Yang (1957). The absorption or attenuation

$(k_i c/(\sqrt{2}\omega))$ for our model, instead, firstly increases up to $h \sim 1$, depending upon the B values, then starts to decrease as h increases furthermore (Chu, 2004b). The results presented here also show the intrinsic thermodynamic properties of the equilibrium states corresponding to the final equilibrium state after the collision of dilute hard-sphere Bose ($B > 0$), Boltzmann ($B = 0$), and Fermi ($B < 0$) gases.

At low temperatures, the Pauli exclusion principle forces Fermi-gas particles to be farther apart than the range of the collisional interaction, and they therefore cannot collide and re-thermalize. That is to say, identical fermions are unable to undergo the collisions necessary to re-thermalize the gas during evaporation because of the need to maximize Pauli blocking efects (Chu, 2002, 2003). The much more spreading characteristics of dispersion relations for dilute Fermi gases ($B < 0$) thus obtained (Chu, 2004b) seems to confirm above theoretical reasoning.

Considering the case of nonzero electric fields, i.e., RHS $\neq 0$, we can obtain the detailed mathematical expression for RHS by following the verified approaches (Ustinov and Kravtsov, 1995; Butler *et al.*, 2000; Noce and Cuoco, 2000) with

$$\mathrm{RHS} \equiv i\frac{e|\mathbf{E}|}{m^*}\delta(\epsilon - \mu_0)c\cos\left(\frac{m-1}{n}\pi\right)\left[c\cos\left(\frac{m-1}{n}\pi\right)k + \omega\right], \quad (8)$$

where δ is the delta function. To obtain similar dispersion relations together with the Eqs. (6) or (7) with nonzero RHS, we must impose the other condition from the Eq. (8) with RHS being zero for arbitrary $\mathcal{C}$. Under this situation, we have anomalous results : $|\lambda_r| = 1/\sqrt{2}$ (the relative phase speed λ_r is negative! cf. (Caldwell, 2002) for similar negative sound speed results considering the phantom energy states characterized by a super-negative equation of state) and $\lambda_i = 0$ for all the rarefaction measure (hs) and the Pauli-blocking parameter (Bs). This strange behavior for $\nu = 1/2$ ($\mathcal{B}_{\text{eff}} = 0$, the electric field ($\mathbf{E}$) effect is being considered) within the composite fermion formulation, however, is similar to that reported in (Chu, 2002, 2003) for the specific case of sound propagating in normal fermionic gases (the Pauli-blocking parameter $B = -1$) or sound propagating in dilute gases (for all Bs but with a free orientation parameter being $\pi/4$). There is no attenuation for above-mentioned cases. This last observation might be relevant to the found *enhanced* conductivity (for 2D electron gases) corresponding to the even-denominator factor $\nu = 1/2$ (composite fermions) using surface acoustic waves (of wavelength smaller than 1 μm) (Willett *et al.*, 1998; Willett and Pfeiffer, 1996) (geometric resonance of the composite fermions' cyclotron orbit and the ultrasound wavelength was also observed at smaller wavelength therein). On the other hand, if we replace the electric field in Eqs. (1), (4) with the gravitational field, then our results (with nonzero external field) of anomalous dispersion relations will qualitatively resemble that reported in (Caldwell, 2002) for similar negative sound speed or super-negative equation of state.

To conclude in brief, by using the quantum discrete kinetic approach, for the case of nonzero electric field, we obtain strange dispersion relations for waves propagating in CF gases with $\nu = 1/2 : |\lambda_r| = 1/\sqrt{2}$ (λ_r is negative) and $\lambda_i = 0$ for all the rarefaction measure (hs) and the Pauli-blocking parameter (Bs). We shall investigate other interesting issues (Ghirardi and Marinatto, 2004; Avancini and Krein, 1995; Harju *et al.*, 2002; Braun and Vechernin, 2004; Kuze and Sirois, 2003; Yabu *et al.*, 2004; Schrieffer, 2004; Agop *et al.*, 2003; Ichinose and Matsui, 2003; Fradkin *et al.*, 1998) in the future.

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